1 Orthotropic Masonry 2D

The material model *Orthotropic Masonry 2D* is an elasto-plastic model, which in addition allows for material softening, possibly different in both local *x* and *y* directions of a surface, under plane-stress condition, especially suited to model (unreinforced) masonry walls. The total strain tensor ε is additively decomposed into the elastic and inelastic parts $\varepsilon = \varepsilon_{el} + \varepsilon_p$. The damage is assumed to follow a smeared crack approach, when the material remains continuum even after damage.

1 Tension

In tension, exponential softening is considered under a Rankine-type yield hypothesis, namely, the yield surface F_t is described as

$$F_t(\boldsymbol{\sigma}, \kappa) = \frac{(\sigma_x - \bar{\sigma}_{t,x}) + (\sigma_y - \bar{\sigma}_{t,y})}{2} + \sqrt{\left(\frac{(\sigma_x - \bar{\sigma}_{t,x}) - (\sigma_y - \bar{\sigma}_{t,y})}{2}\right)^2 + \alpha\tau_{xy}^2}, \tag{1.1}$$

where α controls the amount of shear stress contribution to failure[1], and the back stresses $\bar{\sigma}_{t,i}$ follow an exponential softening law, see Figure 1.1, described by

$$\bar{\sigma}_{t,i}(\kappa) = f_{t,i} \exp\left(-f_{t,i}\frac{h}{G_{t,i}}\kappa\right), \tag{1.2}$$

where $h = \sqrt{A}$ is the equivalent length of the finite element of area A, and $G_{t,i}$ the specific fracture energy (per unit area), i.e., the area under the $\bar{\sigma}$–κ graph.

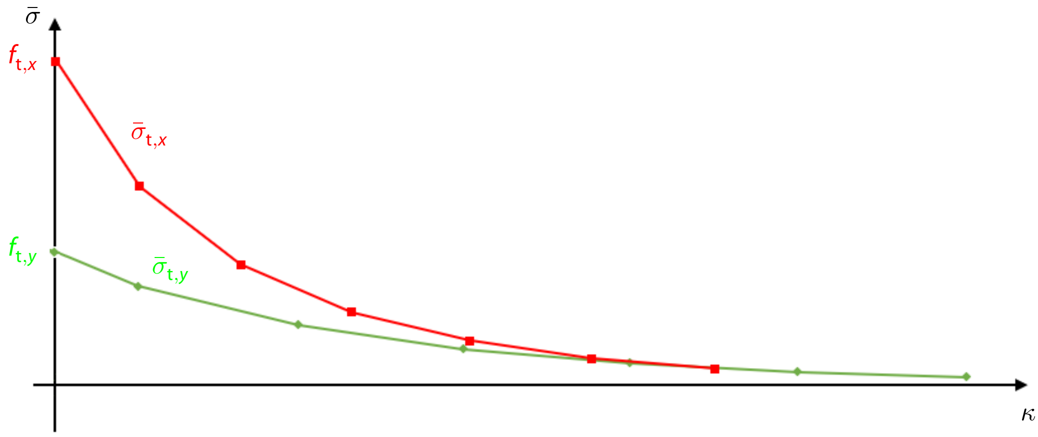

Figure 1.1: Different tensile stress – equivalent strain diagram for *x* and *y* directions of a plate.

The plastic behavior is driven by the maximum principal inelastic strain

$$\dot{\kappa} = \dot{\varepsilon}_{p,1} = \frac{\dot{\varepsilon}_{p,x} + \dot{\varepsilon}_{p,y}}{2} + \frac{1}{2}\sqrt{(\dot{\varepsilon}_{p,x} - \dot{\varepsilon}_{p,y})^2 + (\dot{\gamma}_{p,xy})^2}\,. \tag{1.3}$$

2 Compression

The compression behavior is described by isotropic parabolic hardening (same inelastic strain κ_p at maximum compressive stress) followed by anisotropic parabolic / exponential softening controlled by the compressive fracture energies along the material axes, cf. Figure 1.2, driven by a Hill-type yield criterion—a rotated centered ellipsoid in the plane stress space $(\sigma_x, \sigma_y, \tau_{xy})$—more precisely,

[1] If τ_u denotes pure shear stress, then actually $\alpha = f_{t,x}f_{t,y}/\tau_u^2$. For $\alpha = 1$, the yield criterion $F_t(\boldsymbol{\sigma}, 0) = \sigma_1$ equals the first principal stress.

$$F_{\mathrm{c}}(\boldsymbol{\sigma},\kappa)=\frac{\sigma_x^2}{\bar{\sigma}_{\mathrm{c},x}^2}+\frac{\beta\sigma_x\sigma_y}{\bar{\sigma}_{\mathrm{c},x}\bar{\sigma}_{\mathrm{c},y}}+\frac{\sigma_y^2}{\bar{\sigma}_{\mathrm{c},y}^2}+\frac{\gamma\tau_{xy}^2}{\bar{\sigma}_{\mathrm{c},x}\bar{\sigma}_{\mathrm{c},y}}-1, \tag{1.4}$$

where the additional parameters β, γ are related to the coupling between the normal stresses (mathematically, rotation of the yield surface around the shear axis) and the shear contribution in compression, respectively.[2]

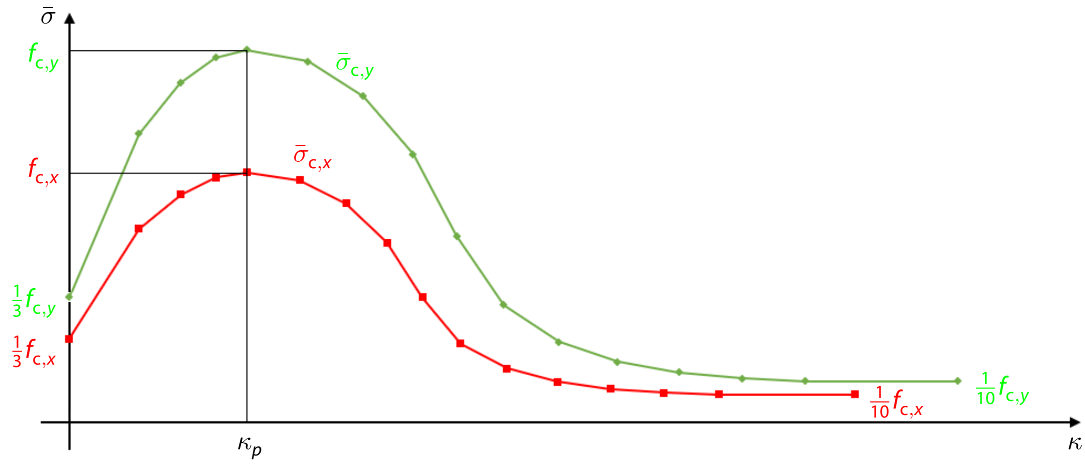

Figure 1.2: Compressive hardening/softening for x and y directions.

An associated flow rule with hardening/softening hypothesis—related to the specific inelastic work—is considered, namely,

$$\dot{\kappa}=\frac{1}{\bar{\sigma}_{\mathrm{c},x}\bar{\sigma}_{\mathrm{c},y}}\boldsymbol{\sigma}^{\top}\dot{\boldsymbol{\varepsilon}}_{\mathrm{p}}. \tag{1.5}$$

1 Material parameter identification

In addition to an orthotropic linear elastic material, there are 7 strength parameters ($f_{\mathrm{t},x}$, $f_{\mathrm{t},y}$, $f_{\mathrm{c},x}$, $f_{\mathrm{c},y}$, α, β, γ) and 5 inelastic parameters ($G_{\mathrm{t},x}$, $G_{\mathrm{t},y}$, $G_{\mathrm{c},x}$, $G_{\mathrm{c},y}$, κ_p) required for the *Orthotropic Masonry 2D* material model in RFEM 5, see Figure 1.3.

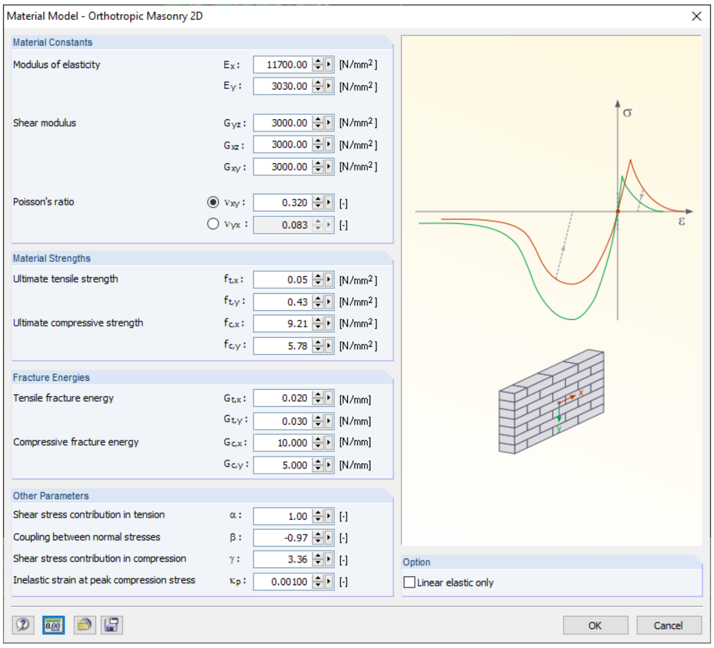

Figure 1.3: Input parameters for *Orthotropic Masonry 2D*.

[2] Analogously to the parameter α for tension, there is $\gamma=f_{\mathrm{c},x}f_{\mathrm{c},y}/\tau_{\mathrm{u}}^2$, while, if f_{45° is the ultimate strength of the material in biaxial compression, then the coupling between normal stresses reads as $\beta=\left(1/f_{45^\circ}^2-1/f_{\mathrm{c},x}^2-1/f_{\mathrm{c},y}^2\right)f_{\mathrm{c},x}f_{\mathrm{c},y}$.

As described in [1], one possible way to obtain these elastic parameters is from the following proposed uniaxial tension/compression tests, see Figure 1.4, and biaxial tests, see Figure 1.5. When performed in a displacement controlled environment, the fracture energies and compressive peak strain are also identifiable.

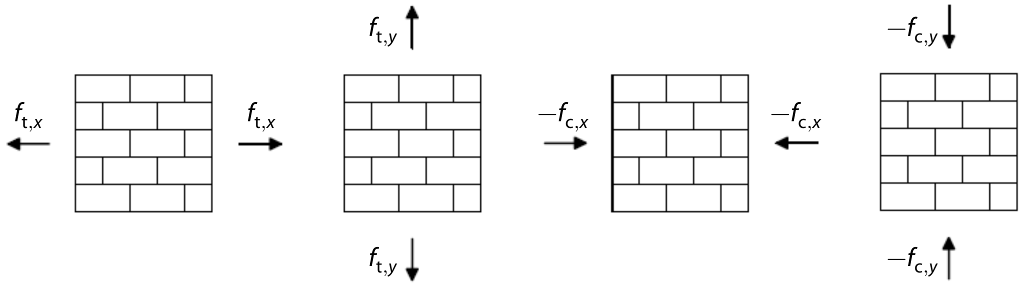

Figure 1.4: Uniaxial masonry tests.

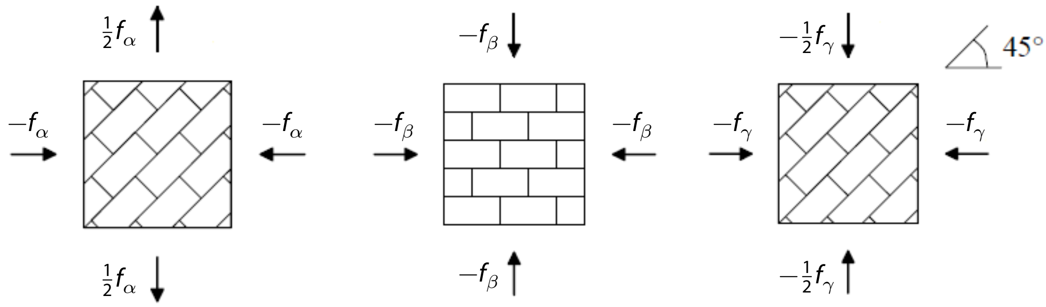

Figure 1.5: Biaxial masonry tests.

Under such conditions α, β, γ read

$$\alpha = \frac{1}{9}\left(1 + 4\frac{f_{t,x}}{f_\alpha}\right)\left(1 + 4\frac{f_{t,y}}{f_\alpha}\right), \tag{1.6}$$

$$\beta = \left(\frac{1}{f_\beta^2} - \frac{1}{f_{c,x}^2} - \frac{1}{f_{c,y}^2}\right) f_{c,x} f_{c,y}, \tag{1.7}$$

$$\gamma = \left[\frac{16}{f_\gamma^2} - 9\left(\frac{1}{f_{c,x}^2} + \frac{\beta}{f_{c,x} f_{c,y}} + \frac{1}{f_{c,y}^2}\right)\right] f_{c,x} f_{c,y}. \tag{1.8}$$

A typical yield surface for the proposed anisotropic Rankine–Hill-type failure cirterion looks as in Figure 1.6.

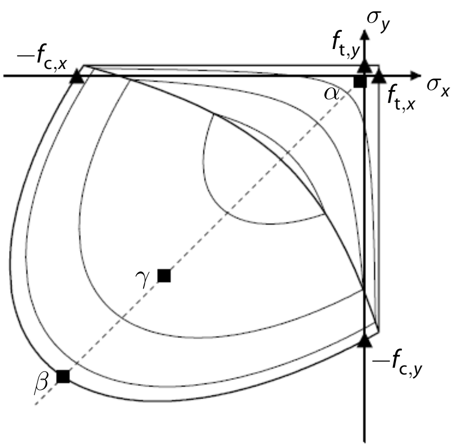

Figure 1.6: Yield surface with iso-shear contour lines and material parameters for *Orthotropic Masonry 2D*.

Literature

[1] P. B. Lourenço. *Computational Strategies for Masonry Structures (PhD diss.)*. Delft University Press, 1996.